OC3140
HW/Lab 5 Sampling Distribution

1. A manufacturer of car batteries guarantees that his batteries will last, on the average, 3 years with a standard deviation of 1 year. If five of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5 and 4.2 years, is the manufacturer still convinced that his batteries have a standard deviation of 1 year? (using 5 %)

Solution:

Try Chi square $\left(c^2\right)$ distribution (see Chapter 5, Page 8)

$x = 1.9, 2.4, 3.0, 3.5, 4.2$, n=5, $s = 1$ (standard deviation of 1 year).

$$\overline{x} = 3.0,\ s^2 = \frac{\sum\left(x_i - \overline{x}\right)^2}{n-1} = 0.815,\ c^2 = \frac{\left(n-1\right)s^2}{s^2} = 3.26.$$

From c^2 distribution table (Chapter 5 Page 12) with

$df = n - 1 = 4$, $P\left(0.711\right) = 0.95$, $P\left(9.488\right) = 0.05$.

Since

$0.711 < c^2\left(= 3.26\right) < 9.488$,

the standard deviation of 1 year is reasonable.

2. A manufacturer of light bulbs claims that his bulbs will burn on the average 500 hours. To maintain this average, he tests 25 bulbs each month. If the computed t-value falls between $-t_{0.05}$ and $t_{0.05}$, he is satisfied with his claim. What conclusion should he draw from a sample that has a mean ($\overline{x} = 518$ hours) and a standard deviation $s = 40$ hours?

Solution:

Compute the t-value (see Chapter 5, Page 13)

$m = 500$, $n = 25$, $\overline{x} = 518$ and $s = 40$.

$$t = \frac{\overline{x} - m}{s/\sqrt{n}} = 2.25.$$

Use the t-distribution table (Chapter 5 Page 17),

$df = n - 1 = 24$, $t_{0.05} = 1.711$.

Since $t = 2.25 > t_{0.05}$, the average hours may be more than 500.

3. For the F distribution find,

   (a) $f_{0.05}$ with $n_1 = 7$ and $n_2 = 15$;
   (b) $f_{0.05}$ with $n_1 = 15$ and $n_2 = 7$;
   (c) $f_{0.05}$ with $n_1 = 24$ and $n_2 = 19$.

Solution:

Using the F distribution Table (see Ch.5 p18 , p21-22)

(a). $f_{0.05}(7,15) = 2.71$
(b). $f_{0.05}(15,7) = 3.51$
(c). $f_{0.01}(24,19) = 2.92$